Extra Credit Problem 1: Calculation of the distribution of different forms of phosphate in a 0.01 M solution at pH 7.0. Use the values 2.2, 7.2 and 12.4 for pK₁, pK₂ and pK₃ respectively.

You have the following four equations in four unknowns:

1. \[
\frac{[H_2PO_4^-]}{[H_3PO_4^-]} = 10^{pH-pK_1} = 10^{7.0-2.2} = 10^{4.8} = 6.31 \times 10^4
\]
\[
[H_2PO_4^-] = [H_3PO_4^-] \times 6.31 \times 10^4
\]

2. \[
\frac{[HPO_4^{2-}]}{[H_2PO_4^-]} = 10^{pH-pK_2} = 10^{7.0-7.2} = 10^{-0.2} = 0.631
\]
\[
[HPO_4^{2-}] = [H_2PO_4^-] \times 0.631
\]

3. \[
\frac{[PO_4^{3-}]}{[HPO_4^{2-}]} = 10^{pH-pK_3} = 10^{7.0-12.4} = 10^{-5.4} = 3.98 \times 10^{-6}
\]
\[
[PO_4^{3-}] = [HPO_4^{2-}] \times 3.98 \times 10^{-6}
\]

4. \[
[H_3PO_4] + [H_2PO_4^-] + [HPO_4^{2-}] + [PO_4^{3-}] = 0.01 M
\]

Substitute 1 for \([H_2PO_4^-]\) in 2:

2. \[
[HPO_4^{2-}] = [H_3PO_4] \times 6.31 \times 10^4 \times 0.631
\]

Substitute 2 for \([HPO_4^{2-}]\) in 3:

3. \[
[PO_4^{3-}] = [H_3PO_4] \times 6.31 \times 10^4 \times 0.631 \times 3.98 \times 10^{-6}
\]

Substitute 1, 2, and 3 into 4 to get 4 as a function of \([HPO_4^{2-}]\):

4. \[
[H_3PO_4] + [HPO_4^{2-}] \times 6.31 \times 10^4
\]

Factor:

\[
[H_3PO_4] \times (1 + 6.31 \times 10^4 + 6.31 \times 10^4 \times 0.631 + 6.31 \times 10^4 \times 0.631 \times 3.98 \times 10^{-6}) = 0.01 M
\]

\[
[H_3PO_4] \times (1.02917 \times 10^5) = 0.01 M
\]

\[
[H_3PO_4] = \frac{0.01 M}{1.02917 \times 10^5} = 9.71654 \times 10^{-8} M
\]

Then plug this value into equations 1, 2, and 3:

\[
[H_2PO_4^-] = 9.71654 \times 10^{-8} M \times 6.31 \times 10^4 = 6.1311 \times 10^{-3} = 0.0061311 M
\]

\[
[HPO_4^{2-}] = 9.71654 \times 10^{-8} M \times 6.31 \times 10^4 \times 0.631 = 3.86875 \times 10^{-3} = 0.00386875 M
\]

\[
[PO_4^{3-}] = 9.71654 \times 10^{-8} M \times 6.31 \times 10^4 \times 0.631 \times 3.98 \times 10^{-6} = 1.53976 \times 10^{-8} M
\]

Check the answers by plugging into equation 4:

\[
9.71654 \times 10^{-8} + 0.0061311 + 0.00386875 + 1.53976 \times 10^{-8} = 9.999996 \times 10^{-3} or 0.01
\]

I carried this out to way too many significant figures to make a point. Actually one should round off for the two major species:

\[
[H_2PO_4^-] = 0.0061311 M \text{ and } [HPO_4^{2-}] = 0.00386875 M
\]

And recognize that the minor species are well below significant concentrations, yet there is some of each species present.