Enzyme Kinetics

Enzyme Characteristics

- Catalytic Power
  - Rate enhancements as much as $10^{14}$
- Specificity
  - Enzymes can distinguish between closely related chemical species
    - D and L isomers
    - \textit{cis} and \textit{trans} isomers
    - Diastereomers (glucose and galactose)
- Regulation
  - Ability to activate or inhibit enzymes can control which reactions occur and when.

An example of specificity is illustrated by the enzyme \textit{fumarase} which catalyzes addition of water to the double bond of \textit{fumaric acid} to form \textit{L-malic acid}. The \textit{cis} isomer of fumarate, maleic acid, does not work. Neither does D-malic acid, the enantiomer of L-malic acid.

Enzyme Terminology

- Substrates
  - Substances whose reaction is being catalyzed
- Products
  - End products of the catalyzed reaction
- For reversible reaction, designation depends on point of view.
  - $A + B \rightleftharpoons P + Q$
  - $A, B$ products, $P, Q$ reactants, or vice versa

An enzyme is a catalyst, and as such does not influence the \textit{equilibrium} position of the reaction—only its rate. So one can start with $A$ and $B$ and get $P$ and $Q$ formed, but one can also start with $P$ and $Q$ and get $A$ and $B$ formed.
**Enzyme Nomenclature**

- Some names are historical common names
  - Trypsin, chymotrypsin,
- Some have common names related to the substrate(s), usually ending in \textit{ase}.
  - urease, protease, ribonuclease, ATPase, glucose-6-phosphatase
- All enzymes have a \textbf{systematic name} based on international agreement

**Systematic Classification of Enzymes**

1. Oxidoreductases
2. Transferases
3. Hydrolases
4. Lyases
5. Isomerases
6. Ligases

**Oxidoreductases**

- Catalyze transfer of electrons between species (i.e., oxidation and reduction)
  - Some are called \textit{dehydrogenases} in their common name.
  - Some are called \textit{oxidases} or \textit{reductases}.
  - Often \textit{coenzymes} such as NAD, NADP, FAD, or a metal ion is involved in the reaction.
Transferases

- Catalyze transfer of a functional group from one molecule to another
  - e.g., phosphate groups, methyl groups, acyl groups, glycosyl groups, etc.
  - Enzymes catalyzing transfer of a phosphate from ATP to another acceptor are commonly referred to as kinases.

Hydrolases and Lyases

- **Hydrolases** catalyze hydrolysis of a chemical bond
  - Includes proteases, esterases, phosphatases, etc.
- **Lyases** catalyze reversible addition of groups to a double bond.
  - Usually H$_2$O or NH$_3$ added
  - Interconversion of fumaric acid and L-malic acid which we alluded to earlier.

Isomerases and Ligases

- **Isomerases** catalyze conversion of one isomer to another.
  - The only enzyme class in which a single substrate and product is involved.
- **Ligases** catalyze the formation of a chemical bond at the expense of hydrolysis of ATP.

We will encounter a number of examples of ligases later.

**Learn the general classification groups.** As we study new enzymes, be able to fit an enzyme into one of these groups, but you won’t be held for knowing the specific systematic name. We will usually refer to more common names—and sometimes there is more than one!
Coenzymes

- Coenzymes are non-protein substances required for activity, sometimes called cofactors.
- (See Table 14.2)
- Metal ions
  - Some metal ion cofactors are directly involved in the catalytic mechanism. Others are required simply for stabilization of protein structure.
- Organic cofactors
  - Non-amino acid compounds required for catalytic activity

Organic Coenzymes

- Two classes of organic coenzymes
  - Coenzyme cosubstrates
    - Transformed in the reaction, regenerated in another reaction
    - Loosely bound to the enzyme, dissociates during course of reaction
  - Coenzyme prosthetic groups
    - Participates in the reaction, but original structure regenerated during catalytic cycle
    - Tightly bound, sometimes covalently, does not dissociate during course of reaction.

Your textbook does not make this distinction clear between these two classes of coenzymes, yet keeping their differences straight is important in understanding some metabolic processes. As we study new enzymes involving coenzymes, be prepared to classify the type of coenzyme involved. An enzyme lacking its prosthetic group is called an apoenzyme. When its prosthetic group is present, it is called a holoenzyme.

Enzyme Kinetics

- Chemical kinetics refers to the rate of chemical reactions, and things which influence that rate.
- Enzymes as catalysts accelerate the rate, but not the equilibrium position.
- A chemical rate law describes the effect of variables, such as reactant concentration, on the rate of the reaction.
Enzyme Kinetics--Applications

What can kinetics tell us?
- Kinetic studies can help us understand how metabolic pathways are controlled, and the conditions under which an enzyme is active.
- Kinetic studies can yield information about the mechanism of an enzymatic reaction.
- However, kinetic studies can only rule out mechanistic models that do not fit the data. They cannot prove a mechanism.

Reaction Rate

- The rate of a reaction is the slope of a progress curve. (See Figure 14.4)
- For the simple reaction:
  - A → P
  
  \[
  \text{rate} = \frac{d[P]}{dt} \quad \text{or} \quad \frac{d[A]}{dt}
  \]

Reaction Rate Theory:
The Transition State

- For chemical bond breakage and formation to occur, the reactants must go through an intermediate transition state.
- The free energy of the transition state is higher than that of either the reactants or the products. (See Figure 14.1)
- Only a small fraction of the reactants have sufficient energy to achieve this "activation energy", referred to as $\Delta G^\ddagger$. Note that the slope of the progress curve changes with time. Most studies in enzyme kinetics try to deal with the initial rate of a reaction, that is the slope of the curve where $t = 0$. Many things can influence the catalytic ability of an enzyme, including the accumulation of products as well as denaturation of the protein itself. It is only at the initiation of the reaction that one is sure of the amount of active enzyme present and the other conditions of substrate and product concentration.
Reaction Rate Theory: Effect of Temperature

• Increasing the temperature increases the fraction of reactants which have enough energy to achieve the transition state.

• This relationship is expressed in the Arrhenius Equation:
  \[ k = A e^{-\Delta G / RT} \]
  where \( \Delta G \) is the activation energy
  (see Figure 14.5a)

Reaction Rate Theory: Effect of a catalyst

• A catalyst accelerates the rate of a reaction by lowering the activation energy of the reaction. (See Figure 14.5b)

Rate Law

• Expresses relationship between rate and concentration of reactants.
  • e.g. \( v = k[A] \), \( v = k[A]^2 \), \( v = k[A][B] \)

• \( k \) is the rate constant (a proportionality constant between rate and the concentration terms).

• Effect of temperature on reaction rate is an effect on \( k \).
Reaction Order

- The order of a reaction is given by the exponents of the reactant concentrations in the rate law.
  - $v = k[A]^n$
    - $n=0$, zero order (no dependence on $[A]$)
    - $n=1$, first order
    - $n=2$, second order
  - $v = k[A]^2[B]$  
    - second order in A, first order in B, third order overall

Reaction Order, con’t.

- Experimental reaction rate order
  - Determined by experimental measurements.
  - Need not be whole numbers
- Theoretical reaction rate order
  - Interpreted as the molecularity of elementary steps in the reaction.
  - The reaction $A + B \rightarrow P + Q$
    - Would have a molecularity of 2, and a theoretical rate law: $v = k[A][B]$

Effect of Enzyme Concentration on Rate

- First establish the effect of Enzyme concentration on reaction rate.
  - Try to work only in region where $v = k[E]$
Effect of Substrate Concentration on Rate

- For many enzymatic reactions, the experimental rate law is given by the equation for a rectangular hyperbola.

\[ v = \frac{aS}{b + S} \]

Michaelis Menten Rate Law

- At high S (S >>> b)
  - v = a
  - zero order in S
- At low S (S <<< b)
  - v = (a/b)S
  - first order in S
Michaelis Menten Model to Explain Experimental Rate Law

- Terminology of M.M. equation
  - $a$ is called $V_m$, the maximum velocity
    - This is the asymptote of the saturation curve, the maximum velocity that can be achieved at a very high $S$ concentration. It has the units of $v$.
  - $b$ is called $K_m$, the Michaelis constant
    - This is the substrate concentration at which $v$ is one-half of $V_m$. It has the units of $S$.

Michaelis Menten Model

- Michaelis and Menten proposed an enzyme mechanism (a model) to explain the rate law behavior.
- Features of their model give some insight into what is happening, nevertheless their model was overly simplistic.
- Deriving the rate law from their model shows it is consistent with the data, but does not prove the model.

Postulates (Assumptions) of the M.M. Model

1. Enzyme and substrate combine to form an ES complex, which breaks down to form product.

\[ E + S \xrightleftharpoons[k_1]{k_{-1}} ES \xrightarrow[k_2]{k_3} E + P \]
Postulates (Assumptions) of the M.M. Model, con’t.

2. $[S] \gg [E]$, so $[S_\text{f}] = [S_{\text{free}}]$
   And $[ES]$ can be ignored
3. Last step is irreversible
   either $k_2 = 0$, or $[P] = 0$
4. $k_2 << k_1$, the rapid equilibrium assumption
   (The binding of $S$ to $E$ is rapid compared to the breakdown of the ES complex.)

Derivations from the M.M. Postulates

(a) $K = \frac{[E][S]}{[ES]} = \frac{k_1}{k_1}$  
(b) $v = k_2 [ES]$

(c) $[E]_\text{f} = [E] + [ES]$
   where $[E]$ is the free enzyme

• Algebraically eliminate $[E]$ and $[ES]$ from these equations to give an equation with $v$
as a function of $[S]$

Derivations from the M.M. Postulates, con’t.

• Divide (b) by (c), and substitute (a)

\[
\frac{v}{[E]_\text{f}} = \frac{k_2 [ES]}{[E] + [ES]} \quad \frac{v}{K} = \frac{k_2 [ES]}{K [ES] + [ES]}
\]

\[
v = \frac{k_2 [E]_\text{f} [S]}{K + [S]} = \frac{V_m [S]}{K + [S]}
\]

where $V_m = k_2 [E]_\text{f}$

and $K = \frac{k_1}{k_i}$
Briggs-Haldane Refinement: The Steady State Assumption

• Assume that [ES] does not change over the course of the reaction.
  • i.e., it is being broken down as rapidly as it is being made. (See Figure 14.8)
  \[
  \frac{d[ES]}{dt} = 0 = k[E][S] - k_{-1}[ES] - k[ES]
  \]
  \[
  k[E][S] = (k_{-1} + k_1)[ES]
  \]
  \[
  \frac{[E][S]}{[ES]} = \frac{k_{-1} + k_1}{k_1} = K_m
  \]

Note the key difference between the results of the rapid equilibrium assumption and the steady state assumption is the interpretation of $K_m$. In the first case, it is the dissociation constant of the ES complex. In the second it is a more complicated assembly of rate constants.

Michaelis-Menten Equation as a Theoretical Rate Law: Summary

\[
v = \frac{V_m [S]}{K_m + [S]}
\]

Based on mechanism:

\[
E + S \xrightleftharpoons[k_{-1}]{k_1} ES \xrightarrow{k_2} E + P
\]

where \( V_m = k_2[E] \) and

\[
K_m = \frac{k_{-1}}{k_1} \text{ (rapid equilibrium assumption)} \quad \text{or} \quad K_m = \frac{k_{-1} + k_1}{k_1} \text{ (steady state assumption)}
\]

International units and katals can be expressed even in a crude mixture where the purity of the enzyme or the actual quantity of enzyme protein present are not known. Turnover number, on the other hand, requires that one know the number of moles of enzyme present in the reaction.

Enzyme Units

• Quantity of an enzyme often measured in terms of its catalytic activity.
  • **International Unit**—amount that catalyzes formation of one micromole of product in one minute.
  • **katal**—amount that catalyzes conversion of one mole of substrate to product in one second
  • **turnover number**—substrate molecules converted per enzyme molecule per unit time.
Turnover Number, con’t.

- Called $k_{\text{cat}}$
- Equivalent to $k_2$ in the M.M. equation provided $V_{\text{max}}$ and $[E_i]$ are in the same molar units.
- For example, $V_{\text{max}}$ in mmol·sec$^{-1}$·ml$^{-1}$ and $[E_i]$ in mmol·ml$^{-1}$
- Varies over a wide range (See Table 14.4)

Catalytic Efficiency

- $k_{\text{cat}}$ measures kinetic efficiency, and $K_m$ is inversely related to the binding affinity
- $k_{\text{cat}}/K_m$ measures the catalytic efficiency of an enzyme—how well it has evolved to do its job.
- It is the first order rate constant at low substrate concentration: $v = \frac{k_{\text{cat}} [S]}{[E_i]} K_m$

Catalytic Efficiency, con’t.

- Using the M.M. model and the steady state assumption:
  $$\frac{k_{\text{cat}}}{K_m} = \frac{k_2}{(k_{-1} + k_3)} = \frac{k_1 k_2}{(k_{-1} + k_3)}$$

  when $k_1 \gg k_{-1}$ and the reaction is
  $$\frac{k_{\text{cat}}}{K_m} = \frac{k_1 k_2}{k_2} = k_1$$
  Diffusion Controlled

The reaction can never proceed faster than the time it takes the substrate to bind to the enzyme in the first place. In the extreme where every substrate binding event leads to product, the reaction is said to be diffusion controlled. Theoretically, the maximum rate for these collisions is about $10^8$ sec$^{-1}$ M$^{-1}$. See Table 14.5 for some enzymes that seem to have achieved this “perfection”.
Validity of M.M. Model Assumptions

1. (a) Enzyme and substrate combine to form an ES complex, which breaks down to form product.
   - This basic assumption must be true in any model.
   (b) One substrate, one product:
   - Seldom true; approximation if everything else is held constant

\[ E + S \xrightleftharpoons[k_i]{k_{-i}} ES \xrightarrow{k_2} E + P \]

Validity of M.M. Model Assumptions, con’t.

2. \([S] \gg [E]\), so \([S] = [S_{\text{free}}]\)
   - Usually true in the test tube, not always in the cell.

3. Last step is irreversible
   - Usually only artificially when \([P] = 0\)

4) Rapid equilibrium, or steady state assumption.
   - Steady state assumption is more general

Expanding the M.M. Model Assumptions

- Adding additional steps in the mechanism
- Making the last step reversible
- More than one substrate or product
Additional Steps in the Mechanism

• Suppose there is a “rearrangement” step added to the mechanism

\[ \begin{align*}
E + S &\xrightleftharpoons[k_{-1}]{k_1} ES \xrightarrow[k_{-2}]{k_2} EP \xrightarrow{k_3} E + P \\
\end{align*} \]

The rate law becomes:

\[ v = \frac{k_1 k_3 [E][S]}{k_2 + k_1 + k_3} \]

You can derive this equation if you wish. Just note there are now three forms of the enzyme (E + ES + EP), and there are two steady state assumption statements: \( \frac{d[ES]}{dt} = 0 = k_1 [E][S] + k_{-2} [EP] - (k_1 + k_2) [ES] \) and \( \frac{d[EP]}{dt} = 0 = k_2 [ES] - (k_{-2} + k_3) [EP] \). And \( v = k_3 [EP] \).

Therefore “steady state” kinetics cannot distinguish between mechanisms that just involve different numbers of rearrangement steps. The more complicated the mechanism, the more complicated the interpretation of \( K_m \) in terms of the parameters of the model.

Additional Steps in the Mechanism, con’t.

• This equation still has the same form as the M.M. equation, but the interpretation of \( V_{max} \) and \( K_m \) are different:

\[ \begin{align*}
V_m &= \frac{k_1 k_3 [E]}{k_2 + k_1 + k_3} \\
K_m &= \frac{k_{-2} k_1 + k_1 k_3}{k_1 (k_2 + k_{-2} + k_3)} \\
\end{align*} \]

Making the Last Step Reversible

\[ \begin{align*}
E + S &\xrightleftharpoons[k_{-1}]{k_1} ES \xrightleftharpoons[k_{-2}]{k_2} EP \xrightleftharpoons[k_{-3}]{k_3} E + P \\
\end{align*} \]

the rate law takes the form

\[ v = \frac{a[S] - b[P]}{c + d[S] + e[P]} \]

where \( a, b, c, d, \) and \( e \) are assemblies of rate constants

more complicated, but not impossible, to analyze
More Than One Substrate or Product

• Even simplest mechanism is much more complex. Several pathways are possible:

Ordered Sequence Mechanisms

• Ordered, Single-Displacement. Substrates add in a definite order.

Ordered Sequence Mechanisms

• Ordered, Double-Displacement (Ping-Pong) First substrate is converted to product before second substrate adds. Requires two forms of “free” enzyme.
Random Addition Mechanisms

- A and B can add in any order, P and Q can be released in any order.

Kinetic Consequences of Multi-substrate Mechanisms

- Hold one substrate constant, and vary the other one:
  - M.M. kinetics (rectangular hyperbola) shown only for:
    - Ordered sequence mechanisms
    - Random mechanisms if the rapid equilibrium assumption holds.

Linear Transformations of the M.M. Equation

- Before the days of computers, linear plots of the M.M. equation were used to calculate values for the kinetic constants $V_{max}$ and $K_m$
- Linear plots are still useful in “visualizing” the data.
- Three common linear plots:
  - Lineweaver-Burk plot
  - Haynes-Woolf plot
  - Eadie-Hofstee plot

Mechanisms which give true rectangular hyperbola give straight lines in these plots. Such mechanisms are referred to as “linear” mechanisms. Hence ordered sequence mechanisms are “linear”, while only “rapid equilibrium” random mechanisms are linear. In all cases, the assumption that the reverse reaction is insignificant must be correct for the results to be linear.

Only skim the textbook analysis of two-substrate reactions. Their treatment is incomplete, and an adequate treatment would take more time than we have available.
Lineweaver-Burk Plot

- Take the reciprocal of the M.M. equation, and rearrange it: plot $1/v$ versus $1/[S]$.

\[
\frac{1}{v} = \frac{K_m + [S]}{V_m[S]} \quad \text{or} \quad \frac{1}{v} = \frac{1}{V_m} + \frac{K_m}{V_m} \cdot \frac{1}{[S]}
\]

\[y = mx + b \quad \text{slope} = \frac{K_m}{V_m}; \text{ intercept} = \frac{1}{V_m}\]

see Figure 14.9

Hanes-Woolf Plot

- Multiply Lineweaver-Burk equation through by $[S]$; plot $[S]/v$ versus $[S]$.

\[
\frac{[S]}{v} = \frac{1}{V_m} [S] + \frac{K_m}{V_m}
\]

See Figure 14.10

Eadie-Hofstee Plot

- Multiply Lineweaver-Burk equation through by $v$ and $V_m$; plot $v$ versus $v/[S]$.

\[
V_m = v + K_m \cdot \frac{v}{[S]}
\]

or \[v = V_m - K_m \cdot \frac{v}{[S]}\]

\[y\text{ intercept} = V_m \quad \text{slope} = -K_m\]
Enzyme Inhibition

- Two general classes of inhibitors
  - Reversible— inhibitors can dissociate and be removed through dilution or dialysis
  - Irreversible— inhibitors form a covalent alteration of the enzyme

Reversible inhibition is usually immediate, whereas irreversible inhibition may vary with time of incubation of the enzyme with the inhibitor. Some irreversible inhibitors are called “suicide” inhibitors, in that the enzyme converts the inhibitor into something which covalently binds to it and destroys its activity. Penicillin is an example. See Figure 14.17.

Classes of Inhibition

- Based on reversible inhibition.
- Based on simple M.M. one-substrate model
- Four types: (Your text only describes three)
  1) Competitive
  2) Uncompetitive
  3) Pure noncompetitive
  4) Mixed noncompetitive
    (2, 3, and 4 also called “not-competitive”)

Competitive Inhibition

- Assumes that the inhibitor binds only to the free enzyme.

\[ E + S \xrightarrow{k_{-1}} E + P \]

\[ E + I \xrightarrow{k_{-2}} EI \quad K_i = \frac{[E][I]}{[EI]} \]

Gives the following rate law:

\[ \frac{1}{v} = \frac{1}{V_m} + \frac{K_i}{V_m} \left( \frac{1}{1 + \frac{[I]}{K_i}} \right) \frac{1}{[S]} \]
Competitive Inhibition, con’t.

- Effect is only on slope and not the intercept of the Lineweaver-Burk plot.
  - See Figure 14.13
  - Interpretation is that the inhibitor binds at the same site as the substrate. Both cannot bind to the free enzyme.
  - Example is succinate dehydrogenase, inhibited by malonate.

Uncompetitive Inhibition

- Assumes that the inhibitor binds only to the ES complex.
  \[ E + S \xrightleftharpoons[k_{-1}]{k_1} ES \xrightarrow{k_2} E + P \]
  \[ ES + I \xrightarrow{k_{-2}} ESI; \quad K'_i = \frac{[ES][I]}{[ESI]} \]
  gives the following rate law:
  \[ \frac{1}{v} = \frac{1}{V_m} \left(1 + \frac{[I]}{K'_i}\right) + \frac{K_m}{V_m [S]} \]

Uncompetitive Inhibition, con’t.

- Effect is on intercept and not slope of the Lineweaver-Burk plot. Gives parallel lines.
- Interpretation is that inhibitor can only bind after substrate has bound
Noncompetitive Inhibition

- Assumes inhibitor binds to either the free enzyme, or the ES complex.

\[ E + S \overset{k_1}{\underset{k_{-1}}{\rightleftharpoons}} ES \overset{k_i}{\longrightarrow} E + P \]

\[ E + I \overset{k_i}{\underset{k_{-i}}{\rightleftharpoons}} EI; \quad ES + I \overset{k_i}{\underset{k_{-i}}{\rightleftharpoons}} ESI \]

\[ K_i = \frac{[E][I]}{[EI]}, \quad K_i' = \frac{[ES][I]}{[ESI]} \]

Noncompetitive Inhibition, con’t.

Gives the following rate law:

\[ \frac{1}{v} = \frac{1}{V_m} \left( 1 + \frac{[I]}{K_i} \right) + \frac{K_m}{V_m} \left( 1 + \frac{[I]}{K_i'} \right) \frac{1}{[S]} \]

- Both intercept and slope of Lineweaver-Burk plot are affected.
- **Pure Noncompetitive**, when \( K_i = K_i' \)
  - Lines cross at x axis. (See Figure 14.15)
- **Mixed Noncompetitive**, when \( K_i \neq K_i' \)
  - Lines do not cross at x axis. (See Figure 14.16)

Ribozymes

Several cases are now known where RNA has catalytic activity like an enzyme.

- RNase P, involved in splicing out a piece of tRNA, has an RNA component.
- Self-splicing RNA from several sources.
- Peptidyl transferase of protein synthesis is an RNA component of the ribosome.
Abzymes

- Enzymes work by lowering the energy of the transition state. They bind the transition state better than the substrate.
- Antibodies raised by using transition state analogues as antigens show catalytic activity.
- So far the reaction types are limited and catalytic enhancement is low.